

One loop renormalization of the non-local gauge invariant
operator $\min_{\{U\}} \int d^4x (A_\mu^a U)^2$ in QCD

J.A. Gracey,
Theoretical Physics Division,
Department of Mathematical Sciences,
University of Liverpool,
P.O. Box 147,
Liverpool,
L69 3BX,
United Kingdom.

Abstract. We compute the one loop anomalous dimension of the gauge invariant dimension two operator $\min_{\{U\}} \int d^4x (A_\mu^a U)^2$, where U is an element of the gauge group, by exploiting Zwanziger's expansion of the operator in terms of gauge invariant non-local n leg operators. The computation is performed in an arbitrary linear covariant gauge and the cancellation of the gauge parameter in the final anomalous dimension is demonstrated explicitly. The result is equivalent to the one loop anomalous dimension of the local dimension two operator $(A_\mu^a)^2$ in the Landau gauge.

The last decade has witnessed an intense interest in the area of dynamical gluon mass generation related to the understanding of the low energy properties of Quantum Chromodynamics (QCD). See, for instance, [1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11]. One motivation for this has been the numerical evidence for the apparent condensation of a dimension two operator to explain the deviation of an effective coupling constant from the expected perturbation theory prediction, [12, 13, 14, 15]. To reconcile the difference one can fit the data more accurately with a $1/Q^2$ correction requiring a dimension two operator to balance the dimensionality of the momentum Q . Ordinarily one would expect a dimension four operator correction as the leading power correction due to the gauge invariant operator $(G_{\mu\nu}^a)^2$ based on the field strength $G_{\mu\nu}^a$ where a is the adjoint colour index. That a dimension two operator appears to emerge in this analysis, [12, 13, 14, 15], is not inconsistent with a variety of other observations made over a period of years. Indeed [16, 17, 18] noted that the perturbative vacuum is unstable and the condensation of a dimension two operator would be energetically favourable. Earlier studies of potential gluon mass operators included a Coulomb gauge analysis of a dimension two operator, [19], as well as Cornwall's construction of a massive gauge invariant QCD Lagrangian which supports vortex solutions, [20, 21]. However, one main theoretical objection to such dimension two operators is that the obvious naive choice, $\frac{1}{2}(A_\mu^a)^2$, where A_μ^a is the gluon field, is clearly gauge variant and therefore not suitable for condensing in quantities involving gauge invariant objects. As the effective coupling constant of [12, 13, 14, 15] is gauge dependent, there is therefore no immediate reason why such a gauge variant operator condensate cannot be the explanation of the $O(1/Q^2)$ power correction. However, phenomenological analyses of gauge invariant quantities, [1], appear to require a gluon mass, albeit tachyonic, to fit experimental data. Motivated by Curci and Ferrari's work from the 1970's, [22], there has been a recent re-examination of $\frac{1}{2}(A_\mu^a)^2$ and its BRST invariant extension since their model possesses a classical gluon mass term. The main drawback of the Curci-Ferrari model for being a possible Lagrangian of a massive vector boson is that the BRST operator is not nilpotent and hence unitarity is absent, [23, 24]. Instead the hope would be that the *quantum* condensation of the operator would circumvent the unitarity objection. Though this is still an open question.

In this respect one calculation of note was the Landau gauge analysis of [4] for the condensation of $\frac{1}{2}(A_\mu^a)^2$ in Yang-Mills theory using the local composite operator method. This was later extended to QCD for massless quarks in [25]. The premise of investigation of [4] rests in the observation that one can in fact have a gauge invariant dimension two operator in QCD which can condense. Indeed the resultant phenomenology of the particular operator in question has been elaborated on in [26]. This operator is given by $\min_{\{U\}} \int d^4x (A_\mu^{aU})^2$, where U is an element of the gauge group which transforms the gauge field along a gauge orbit, and is by nature non-local. Its role in constructing a gauge fixing which is globally consistent and devoid of Gribov ambiguities, [27], has been discussed in, for instance, [28, 29, 30, 31]. That non-locality should play a role in aiming to describe infrared gluon dynamics should come as no surprise in that asymptotic freedom indicates that only at high energies are quarks and gluons effectively free whilst being hard to separate at low energies with lower energy interactions needing to be communicated over large distances. For [4] the main initial technical hurdle to be overcome was the fact that the non-local operator is an infinite coupling constant series in an arbitrary gauge. Hence to do a full perturbative analysis and construct a gauge invariant effective potential was initially impossible. However, by taking the point of view that such an effective potential exists then it seemed sensible to consider it in one gauge. Specifically, the Landau gauge was chosen whence the gauge invariant non-local operator truncates to the one local term $\frac{1}{2}(A_\mu^a)^2$, [4]. Moreover, this operator is renormalizable leading to the successful analysis of the operator's condensation in the two loop effective potential, [4, 25]. Interestingly, the operator in the Landau gauge does not possess an independent renormalization since its anomalous dimension is the sum of

the gluon and Faddeev-Popov ghost anomalous dimensions, [32, 33, 34]. A similar structure is present in the analogous operator in the Curci-Ferrari gauge, [35, 36], and the maximal abelian gauge, [37, 38].

In light of these observations one would still wish to handle the gauge invariant non-local operator itself within the context of quantum field theory with the ultimate aim of constructing a gauge independent effective potential in order to study the condensation, similar to [4, 25]. As a first stage in such an exercise, the main purpose of this article is to renormalize $\min_{\{U\}} \int d^4x (A_\mu^a U)^2$ at one loop in an arbitrary covariant gauge and verify its independence of the gauge parameter as well as the equivalence of its anomalous dimension to the Landau gauge expression. From the nature of the operator this is not a trivial task as one has to renormalize a non-local operator in a gluon 2-point function. Interestingly it transpires that a part of the calculation has already been performed but in a different context, [39, 40], which we naturally adapt for the present work. Before we discuss this it is necessary to indicate the relevant background to the problem and indicate the pitfalls of an initial naive approach. First, we set notation for building the gauge invariant non-local operator by defining

$$\mathcal{O} \equiv \frac{1}{2} \min_{\{U\}} \int d^4x (A_\mu^a U)^2. \quad (1)$$

Given a gauge field $A_\mu = A_\mu^a T^a$, where T^a are the colour group generators, it is transported along a gauge orbit by the (global) gauge transformation

$$A_\mu^U = U A_\mu U^\dagger - \frac{i}{g} (\partial_\mu U) U^\dagger \quad (2)$$

where g is the coupling constant. By construction, the field A_μ^U is gauge invariant and hence it is trivial to see that the operator \mathcal{O} is gauge invariant. However, the explicit form of A_μ^U can be determined by setting

$$U = e^{i\phi^a T^a} \quad (3)$$

where ϕ^a can be deduced order by order in perturbation theory. For instance, [30, 39],

$$\begin{aligned} A_\mu^a U = & \left[\eta_{\mu\nu} - \frac{\partial_\mu \partial_\nu}{\partial^2} \right] \left[A^{a\nu} + g f^{abc} \left(\frac{1}{\partial^2} \partial^\sigma A_\sigma^b \right) A^{c\nu} \right. \\ & \left. - \frac{g}{2} f^{abc} \left(\frac{1}{\partial^2} \partial^\sigma A_\sigma^b \right) \left(\frac{1}{\partial^2} \partial^\nu \partial^\rho A_\rho^c \right) + O(g^2) \right] \end{aligned} \quad (4)$$

where f^{abc} are the colour group structure constants, from which it follows that

$$\begin{aligned} \mathcal{O} = & \frac{1}{2} \int d^4x \left[A_\mu^a \left(\eta^{\mu\nu} - \frac{\partial^\mu \partial^\nu}{\partial^2} \right) A_\nu^a - 2g f^{abc} \left(\frac{1}{\partial^2} \partial^\nu \partial^\sigma A_\sigma^a \right) \left(\frac{1}{\partial^2} \partial^\rho A_\rho^b \right) A_\nu^c \right. \\ & \left. - g f^{abc} A_\nu^a \left(\frac{1}{\partial^2} \partial^\sigma A_\sigma^b \right) \left(\frac{1}{\partial^2} \partial^\nu \partial^\rho A_\rho^c \right) + O(g^2) \right]. \end{aligned} \quad (5)$$

In QED the operator truncates and represents the square of the transverse component of the gauge field. In the non-abelian case the operator involves an infinite number of terms and is assumed to converge.

Naively one would expect to be able to renormalize this version of \mathcal{O} order by order in perturbation theory since at any order there are only a finite number of terms. It transpires that this is not possible without generating several difficult technical problems. First, at one loop one would have to extract the divergent part of the topologies represented in figures 1 and 2 where the encircled cross denotes the location of the operator insertion. Ordinarily to

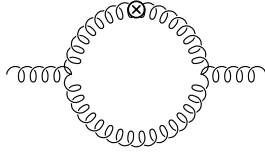


Figure 1: Two leg operator insertion in gluon 2-point function.

renormalize the graph of figure 1 one inserts the operator at zero momentum. However, as is well known, (see, for example, [41]), one can obtain spurious results since the basis of operators into which the Feynman integral decomposes is not closed or complete for this momentum configuration. Therefore, one has to have non-zero momentum operator insertion. Nullifying one of the remaining external momenta, though, results in additional infrared divergences which need to be handled. To circumvent these difficulties one approach is to introduce a spurious infrared regularizing mass which allows for the nullification of external momenta. Whilst this is in principle possible for local operators, for the particular operator we are concerned with it has an inherent non-locality which could lead to further difficulties at higher loops.

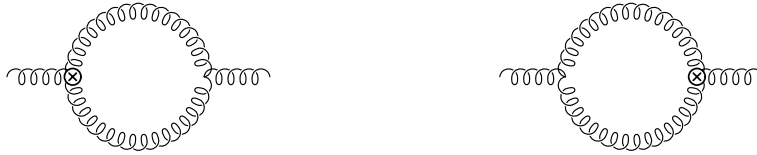


Figure 2: Three leg operator insertion in gluon 2-point function.

Rather than trying to handle these technical issues, it seems more appropriate to regard \mathcal{O} in a different way and use current results to extract its anomalous dimension. In [30] the gauge invariant operator \mathcal{O} was rewritten as the sum of gauge invariant operators which can be treated individually in the renormalization procedure. Specifically, writing the summation of (5) in this non-perturbative way, [30], we have

$$\mathcal{O} = \frac{1}{2} \int d^4x \sum_{n=2}^{\infty} \mathcal{O}_n \quad (6)$$

where

$$\mathcal{O}_2 = - \frac{1}{2} G^{a\mu\nu} \frac{1}{D^2} G^{a\mu\nu} \quad (7)$$

and, [30],

$$\begin{aligned} \mathcal{O}_3 = & g f^{abc} \left(\frac{1}{D^2} G^{a\mu\nu} \right) \left(\frac{1}{D^2} D^\sigma G_{\sigma\mu}^b \right) \left(\frac{1}{D^2} D^\rho G_{\rho\nu}^c \right) \\ & - g f^{abc} \left(\frac{1}{D^2} G^{a\mu\nu} \right) \left(\frac{1}{D^2} D^\sigma G_{\sigma\rho}^b \right) \left(\frac{1}{D^2} D^\rho G_{\mu\nu}^c \right) \end{aligned} \quad (8)$$

with the covariant derivative, D_μ , and field strength, $G_{\mu\nu}^a$, given by

$$D_\mu A_\nu^a = \partial_\mu A_\nu^a - g f^{abc} A_\mu^b A_\nu^c, \quad G_{\mu\nu}^a = \partial_\mu A_\nu^a - \partial_\nu A_\mu^a - g f^{abc} A_\mu^b A_\nu^c. \quad (9)$$

Indeed one could regard this as an expansion in operators with n legs where n is determined from the lowest number gluon legs in each sub-operator \mathcal{O}_n .

The first term (7) of (6) has already been studied in depth in [39, 40] where it was considered as a potential alternative mass operator for the gluon which was gauge invariant though non-local. Its anomalous dimension has been computed to two loops in the $\overline{\text{MS}}$ scheme in an arbitrary linear covariant gauge and is given by, [39, 40],

$$\begin{aligned} \gamma_{\mathcal{O}_2}(a) = & -\frac{1}{3}[11C_A - 4T_F N_f]a - \frac{\lambda^{abcd}\lambda^{abcd}}{128N_A} + f^{abe}f^{cde}\lambda^{adb}\frac{a}{8N_A} \\ & + \left[\frac{77}{12}C_A^2 - \frac{4}{3}C_A T_F N_f - 4C_F T_F N_f\right]a^2 + O(a^3) \end{aligned} \quad (10)$$

in terms of $a = g^2/(16\pi^2)$ where

$$\text{tr}(T^a T^b) = T_F \delta^{ab} \quad , \quad T^a T^a = C_F I \quad , \quad f^{acd}f^{bcd} = C_A \delta^{ab} \quad (11)$$

and N_A is the dimension of the adjoint representation. Clearly, the final expression is independent of the usual gauge parameter. To determine this result, the approach was to first localize \mathcal{O}_2 by introducing a set of additional localizing fields and associated (anti-commuting) ghost fields in such a way as to produce a renormalizable operator, [39, 40]. From the algebraic renormalizability analysis to ensure a multiplicatively renormalizable localization additional quartic interactions between all the localizing fields with couplings, λ^{abcd} , are required. These appear for the first time at two loops in the operator anomalous dimension, (10). Therefore, in the context of determining the anomalous dimension of \mathcal{O} the contribution from the first term of (6) is known. As we are only concerned with one loop, the piece involving the quartic couplings will not become relevant before any two loop renormalization. Hence, to complete the calculation of the anomalous dimension of \mathcal{O} , $\gamma_{\mathcal{O}}(a)$, all that is required is the piece deriving from (8) whose contributions will be deduced from the graphs of figure 2. These give $\gamma_{\mathcal{O}_3}(a)$ whence $\gamma_{\mathcal{O}}(a)$ emerges at one loop from the sum of $\gamma_{\mathcal{O}_2}(a)$ and $\gamma_{\mathcal{O}_3}(a)$.

At one loop this is actually a simple calculation primarily as a result of the topology of the two graphs of figure 2. Since the operator connects to an external leg, the problem of whether a zero or non-zero momentum actually flows through the operator insertion does not arise. The net flow through the combination of external leg and connecting operator is non-zero which can be distributed across both with neither being zero. In other words none of the earlier external momenta nullification complications arising in the two leg insertion of figure 1 will arise for the graphs of figure 2. Moreover, the non-locality resident in the operator does not lead to any additional difficulties and the Feynman rule is simple to derive. This is due to the fact that the operator $1/D^2$ can be replaced by $1/\partial^2$ at one loop and this only acts on *one* field for the leading leg term. Therefore, we have computed the two graphs of figure 2 with \mathcal{O}_3 inserted in an arbitrary covariant gauge and found that the sum of the contributions to $\gamma_{\mathcal{O}}(a)$ from both graphs is

$$\gamma_{\mathcal{O}_3}(a) = \frac{3}{4}C_A a + O(a^2) \quad (12)$$

independent of the gauge parameter. For this particular calculation, we used the MINCER algorithm for the evaluation of massless 2-point Feynman diagrams, [42, 43], written in the symbolic manipulation language FORM, [44], where the electronic representation of the graphs were generated via the QGRAF package, [45], before being converted to FORM input notation. Moreover, we use dimensional regularization in $d = 4 - 2\epsilon$ dimensions and absorb the divergences into the renormalization constants using the $\overline{\text{MS}}$ scheme. The validity of using the MINCER algorithm for massless propagators follows because the Feynman integrals are infrared safe and no infrared regularizing mass needs to be introduced. The respective numerical contributions to (12) from the two operators of (8) are $33C_A/4$ and $-15C_A/2$, which are each independent of

the gauge parameter since each operator is itself gauge invariant. Thus adding all contributions, we find

$$\gamma_{\mathcal{O}}(a) = -\frac{1}{12}[35C_A - 16T_F N_f]a + O(a^2) \quad (13)$$

where the independence of the gauge parameter is a trivial consequence. Moreover, the result is in agreement with the anomalous dimension of $\frac{1}{2}(A_\mu^a)^2$ computed in the Landau gauge, [30, 31, 32]. Additionally the result appears to be consistent with the expectation that the anomalous dimension of a *local* gauge invariant operator is gauge independent, even though the operator itself is *non-local*.

We conclude by noting that it would appear that one can renormalize the non-local operator \mathcal{O} order by order in perturbation theory using Zwanziger's non-perturbative decomposition into gauge invariant non-local operators. However, a more stringent check of this would require a full *two* loop calculation which is a non-trivial exercise. Although the contribution to the overall anomalous dimension from the localized version of the two leg part of the operator is already available, [39, 40], much of the groundwork has to be developed for the three and four leg operator insertions, such as the Feynman rules. Moreover, the four leg gauge invariant operator has yet to be constructed. In addition, in order to apply the MINCER algorithm it is not inconceivable that both \mathcal{O}_3 and \mathcal{O}_4 would need to be localized first by extending the original algebraic renormalization analysis of \mathcal{O}_2 , [39, 40]. Nevertheless, such a calculation would be interesting since it could validate the observation that the full anomalous dimension of \mathcal{O} is in fact given by the Landau gauge value which in turn is determined from the sum of the gluon and Faddeev-Popov ghost anomalous dimensions in that specific gauge.

Acknowledgement. The author acknowledges useful discussions with D.J. Broadhurst, R.E. Browne, D. Dudal, D. Kreimer, S.P. Sorella and H. Verschelde. Support from the Erwin Schrödinger Institute, Vienna, Austria is also gratefully acknowledged as well as the organisers of the Mathematical and Physical Aspects of Perturbative Approaches to Quantum Field Theory programme during which part of this work was carried out.

References.

- [1] K.G. Chetyrkin, S. Narison & V.I. Zakharov, Nucl. Phys. **B550** (1999), 353.
- [2] K.-I. Kondo & T. Shinohara, Phys. Lett. **B491** (2000), 263.
- [3] M. Schaden, hep-th/9909011.
- [4] H. Verschelde, K. Knecht, K. van Acoleyen & M. Vanderkelen, Phys. Lett. **B516** (2001), 307.
- [5] K.-I. Kondo, Phys. Lett. **B514** (2001), 335.
- [6] M. Schaden, hep-th/0108034.
- [7] K.-I. Kondo, hep-ph/0110013.
- [8] K.-I. Kondo, T. Murakami, T. Shinohara & T. Imai, Phys. Rev. **D65** (2002), 085034.
- [9] U. Ellwanger & N. Wschebor, Int. J. Mod. Phys. **A18** (2003), 1595.
- [10] K.-I. Kondo & T. Imai, hep-th/0206173.

- [11] D. Dudal & H. Verschelde, J. Phys. **A36** (2003), 8507.
- [12] P. Boucaud, G. Burgio, F. di Renzo, J.P. Leroy, J. Micheli, C. Parrinello, O. Pène, C. Pittori, J. Rodriguez-Quintero, C. Roiesnel & K. Sharkey, JHEP **04** (2000), 006.
- [13] P. Boucaud, A. Le Yaouanc, J.P. Leroy, J. Micheli, O. Pène & J. Rodriguez-Quintero, Phys. Rev. **D63** (2001), 114003.
- [14] P. Boucaud, F. de Soto, J.P. Leroy, A. Le Yaounac, J. Micheli, H. Moutarde, O. Pène & J. Rodriguez-Quintero, hep-lat/0504017.
- [15] P. Boucaud, J.P. Leroy, A. Le Yaounac, A.Y. Lokhov, J. Micheli, O. Pène, J. Rodriguez-Quintero & C. Roiesnel, JHEP **01** (2006), 037.
- [16] G. Savvidy, Phys. Lett. **B71** (1977), 133.
- [17] F.V. Gubarev, L. Stodolsky & V.I. Zakharov, Phys. Rev. Lett. **86** (2001), 2220.
- [18] F.V. Gubarev & V.I. Zakharov, Phys. Lett. **B501** (2001), 28.
- [19] J. Greensite & M.B. Halpern, Nucl. Phys. **B271** (1986), 379.
- [20] J.M. Cornwall, Phys. Rev. **D26** (1982), 1453.
- [21] J.M. Cornwall & A. Soni, Phys. Lett. **B120** (1983), 431.
- [22] G. Curci & R. Ferrari, Nuovo Cim. **A32** (1976), 151.
- [23] G. Curci & R. Ferrari, Nuovo Cim. **A35** (1976), 1; Nuovo Cim. **A47** (1978), 555.
- [24] G. Curci & R. Ferrari, Nuovo Cim. **A35** (1976), 273.
- [25] R.E. Browne & J.A. Gracey, JHEP **0311** (2003), 029.
- [26] F.V. Gubarev & V.I. Zakharov, Phys. Lett. **B501** (2001), 28.
- [27] V.N. Gribov, Nucl. Phys. **B139** (1978), 1.
- [28] M.A. Semenov-Tian-Shanskii & V.A. Franke, Zap. Nauchn. Semin. LOMI **120** (1982), 159; J. Sov. Math. **34** (1986), 1999.
- [29] G. Dell'Antonio & D. Zwanziger, Commun. Math. Phys. **138** (1991), 291.
- [30] D. Zwanziger, Nucl. Phys. **B345** (1990), 461.
- [31] C. Parrinello & G. Jona-Lasinio, Phys. Lett. **B251** (1990), 175.
- [32] R.M. Doria, F.A.B. Rabelo de Carvalho & S.P. Sorella, Braz. J. Phys. **20** (1990), 316.
- [33] J.A. Gracey, Phys. Lett. **B552** (2003), 101.
- [34] D. Dudal, H. Verschelde & S.P. Sorella, Phys. Lett. **B555** (2003), 126.
- [35] R.E. Browne, D. Dudal, J.A. Gracey, V.E.R. Lemes, M.S. Sarandy, R.F. Sobreiro, S.P. Sorella & H. Verschelde, J. Phys. **A39** (2006), 7889.
- [36] N. Wschebor, hep-th/0701127.
- [37] D. Dudal, H. Verschelde, V.E.R. Lemes, M.S. Sarandy, R.F. Sobreiro, S.P. Sorella, M. Picariello, J.A. Gracey, Phys. Lett. **B569** (2003), 57.

- [38] D. Dudal, J.A. Gracey, V.E.R. Lemes, M.S. Sarandy, R.F. Sobreiro, S.P. Sorella & H.Vershelde, Phys. Rev. **D70** (2004), 114038.
- [39] M.A.L. Capri, D. Dudal, J.A. Gracey, V.E.R. Lemes, R.F. Sobreiro, S.P. Sorella & H. Vershelde, Phys. Rev. **D72** (2005), 105016.
- [40] M.A.L. Capri, D. Dudal, J.A. Gracey, V.E.R. Lemes, R.F. Sobreiro, S.P. Sorella & H. Vershelde, Phys. Rev. **D74** (2006), 045008.
- [41] J.C. Collins, *Renormalization* (Cambridge University Press, 1984).
- [42] S.G. Gorishny, S.A. Larin, L.R. Surguladze & F.K. Tkachov, Comput. Phys. Commun. **55** (1989), 381.
- [43] S.A. Larin, F.V. Tkachov & J.A.M. Vermaseren, “The Form version of Mincer”, NIKHEF-H-91-18.
- [44] J.A.M. Vermaseren, math-ph/0010025.
- [45] P. Nogueira, J. Comput. Phys. **105** (1993), 279.